## SPHERICAL DOME FORMULAS

## Spheroid Dome

Circumference of base: $C=2 \pi r$
Floor Area:
$F_{a}=\pi r^{2}$
Radius of Curvature: $\quad R_{c}=\frac{r^{2}+h^{2}}{2 h}$
Surface Area:
$S_{a}=2 \pi h R_{c}=\pi\left(h^{2}+r^{2}\right)$
Radius at second level: $r_{l}=\sqrt{R_{c}^{2}-\left(R_{c}-h+l\right)^{2}}$


Volume:
$V_{s}=\frac{1}{3} \pi h^{2}\left(3 R_{c}-h\right)=\frac{1}{6} \pi h\left(3 r^{2}+h^{2}\right)$
Skin Tension: $\quad T_{S}=\frac{P_{a} R_{c}}{2}$
Air Pressure: $\quad P_{a}=1^{\prime \prime}$ water column $=0.0361 p s i=5.2 p s f$

## Explanation of terms

- $\quad \pi-$ is the number Pi (pronounced "pie"). Pi is the distance around the edge of a circle divided by its diameter. For our purposes the number $\pi$ is a constant of 3.14159.
- $d-$ is the diameter of the base of the dome.
- $\quad r-$ is the radius of the base. It is equal to half the diameter.
- $\quad R_{c}$ - is the Radius of Curvature. A spheroid dome is a segment of a sphere. Usually the top or cap of a sphere, but it can be any segment including half the sphere (hemisphere) or greater. It is helpful to think of a dome as a sliced off top of a basketball. The shape is always that of the whole basketball no matter where or how it is cut. The radius of curvature is the distance to the center of the sphere.

Example: 40 foot diameter by 15 foot tall dome. $r=\frac{40}{2}=20$ feet
$R_{c}=\frac{r^{2}+h^{2}}{2 h}=\frac{20^{2}+15^{2}}{2 * 15}=\frac{20 * 20+15 * 15}{30}=\frac{400+225}{30}=\frac{625}{30}=20.83 \mathrm{feet}$

- $\quad l$ - is the distance from the base of the dome to a second level (like a second floor).
- $\quad r_{l}$ - is the radius of the dome at a second level ( $l$ high $)$. This radius is helpful to create a second floor layout or to check clearances for doors and windows. Floor area and circumference at this level is calculated using the same formulas for the whole dome (where $r_{l}$ is substituted for $r$ ).
- $\quad C$ - is the circumference or perimiter of the base of the dome (the distance around the dome).

Example: $40^{\prime} \times 15^{\prime}$ dome $-C=\pi d=3.14159 * 40=125.66$ feet

- $\quad F_{a}$ - is the area of the floor of the dome.

Example: $40^{\prime} \times 15^{\prime}$ dome $-F_{a}=\pi r^{2}=3.14159 * 20^{2}=3.14159 * 20 * 20=1,256 \mathrm{ft}^{2}$

- $\quad S_{a}$ - is the surface area of the dome. (This is the equation where $R_{c}$ is used most often)

Example: 40 ' x $15^{\prime}$ dome $-S_{a}=2 \pi h R_{c}=2 * 3.14159 * 15 * 20.83=1,963 \mathrm{ft}^{2}$

## ELLIPSOID DOME FORMULAS

Ellipsoids are difficult to calculate and understand, however, they make very useful dome shapes. Our most common shape is the oblate ellipsoid. It looks like a standard spherical dome with a circular base, but it is "squashed" a little. The sides are more vertical and the top is flatter. This makes smaller "house" size domes that have a little more headroom along the dome wall. A prolate ellipsoid looks more like a watermelon. It is useful in creating a unique building shape.

Ellipse: (Let $a$ be the semi-major axis and $b$ be the semi-minor axis.)
Elliptical formula: $\quad \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$
Eccentricity: $\quad \in=\frac{\sqrt{a^{2}-b^{2}}}{a}=\sqrt{1-\frac{b^{2}}{a^{2}}}$


Oblate Ellipsoid: An oblate ellipsoid is formed by the rotation of an ellipse about its minor axis. Let $a$ be the semi-major axis and $b$ be the semi-minor axis. Let $\in$ be the eccentricity of the revolving ellipse.

Minimum Semi-minor to Semi-major axis ratio: $1: 1.35$
Surface area for entire oblate ellipsoid:

$$
\begin{aligned}
& S_{o}=\pi a^{2}+\frac{\pi b^{2}}{2 \epsilon} \ln \left(\frac{1+\epsilon}{1-\epsilon}\right) \\
& V_{o}=\frac{4}{3} \pi b a^{2}
\end{aligned}
$$

Volume for entire oblate ellipsoid:

Prolate Ellipsoid: A prolate ellipsoid is formed by the rotation of an ellipse about its major axis. Let $a$ be the semi-major axis and $b$ be the semi-minor axis. Let $\in$ be the eccentricity of the revolving ellipse.

Surface area for the entire prolate ellipsoid:
Volume for entire prolate ellipsoid:

$$
\begin{aligned}
& S_{p}=2 \pi b^{2}+\frac{2 \pi a b \arcsin (\epsilon)}{\epsilon} \\
& V_{p}=\frac{4}{3} \pi a b^{2}
\end{aligned}
$$



Volume forentireprolate

